4.4 Solving Systems of Equations

$1x + 1y = 8$

$2x + y = 1$ $\quad y = -2x + 1$

$$\begin{matrix} & x & y & c \\ \text{Eq1} \\ \text{Eq2} \end{matrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

Augment

$$\left[\begin{array}{cc|c} 1 & 1 & 8 \\ 2 & 1 & 1 \end{array}\right]$$

Elementary Row Operations

1. Swap rows
2. Multiply or divide a row by any number except 0
3. Multiply or divide a row by a number (except 0) add to another row
   Row that add to is new row

Reduced Row Echelon Form

$$\left[\begin{array}{cc|c} x & y & \text{constant} \\ \hline 1 & 0 & 5 \\ 0 & 1 & 2 \end{array}\right]$$

Ex. $x = 5$, $y = 2$

$$\begin{bmatrix} 1 & 1 & 8 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & c_1 \\ 0 & 1 & c_2 \end{bmatrix}$$

$R_1 \cdot -2$
$+ R_2$
New R_2

$$\begin{bmatrix} 1 & 1 & 8 \\ 0 & -1 & -15 \end{bmatrix}$$

$R_2 \cdot -1$
New R_2

$$\begin{bmatrix} 1 & 1 & 8 \\ 0 & 1 & 15 \end{bmatrix}$$

$R_2 \cdot -1$
$+ R_1$
New R_1

$$\begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 15 \end{bmatrix}$$

x y c

$x + 0y = -7$

$0x + y = 15$

$x = -7$
$y = 15$

$(-7, 15)$

$$-x + 2y = 12$$
$$x + 6y = 20$$

1st 4th 2nd 3rd C_1 C_2

$$\left[\begin{array}{cc|c} -1 & 2 & 12 \\ 1 & 6 & 20 \end{array}\right]$$

Swap R_1 & R_2

$$\left[\begin{array}{cc|c} 1 & 6 & 20 \\ -1 & 2 & 12 \end{array}\right]$$

$R_1 + R_2$ New R_2

$$\left[\begin{array}{cc|c} 1 & 6 & 20 \\ 0 & 8 & 32 \end{array}\right]$$

$R_2 \div 8$ New R_2

$$\left[\begin{array}{cc|c} 1 & 6 & 20 \\ 0 & 1 & 4 \end{array}\right]$$

$R_2 \cdot -6$ + R_1 New Row 1

$$\left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 4 \end{array}\right]$$

$x = -4$

$y = 4$

$(-4, 4)$

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12. $\left[\begin{array}{cc|c} -3 & 1 & -3 \\ 6 & -12 & 6 \end{array}\right]$

16 $\left[\begin{array}{ccc|c} 12 & 1 & -1 & -7 \\ 11 & 2 & 0 & -2 \\ -1 & 9 & 0 & -9 \end{array}\right]$

15 $\left[\begin{array}{ccc|c} 0 & 1 & 5 & -14 \\ -2 & 3 & -1 & 2 \\ 6 & 0 & -3 & 21 \end{array}\right]$