

5.4 Completing the Square

$$2x^2 + 6x - 5 = 0$$

$$2x^2 + 6x = 5$$

$$\text{factor out } \frac{1}{2} \left(x^2 + 3x + \frac{9}{4} \right) = 5 + \frac{9}{2}$$

$$\frac{3}{2} \left(\frac{3}{2} \right)^2 \text{ factor}$$

$$2 \left(x + \frac{3}{2} \right) \left(x + \frac{3}{2} \right) = \frac{19}{2}$$

$$2 \left(\frac{9}{4} \right) = \frac{9}{2}$$

$$\frac{1}{2} \cdot 2 \left(x + \frac{3}{2} \right)^2 = \frac{19}{2} \cdot \frac{1}{2}$$

$$\sqrt{\left(x + \frac{3}{2} \right)^2} = \sqrt{\frac{19}{4}}$$

$$x + \frac{3}{2} = \pm \frac{\sqrt{19}}{2}$$

$$x = -\frac{3}{2} \pm \frac{\sqrt{19}}{2}$$

$$\text{OR } x = \frac{-3 \pm \sqrt{19}}{2}$$

$$2x^2 + 8x - 1 = 0$$

$$2x^2 + 8x = 1$$

Factor
Add 2

$$2(x^2 + 4x + 4) = 1 + 8$$

Complete
Square

$$2(x+2)(x+2) = 9$$

$$\frac{4}{2} = 2$$

$$2^2 = 4$$

$$\frac{2(x+2)^2}{2} = \frac{9}{2}$$

$$\sqrt{(x+2)^2} = \sqrt{\frac{9}{2}}$$

$$x+2 = \pm \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Rationalize}$$

$$x+2 = \pm \frac{3\sqrt{2}}{2}$$

$$x = -2 \pm \frac{3\sqrt{2}}{2}$$

$$\text{OR } x = \frac{-4 \pm 3\sqrt{2}}{2}$$

$$3x^2 - 2x - 12 = 0 \quad +12 \quad +12$$

$$3x^2 - 2x = 12$$

$$3\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) = 12 + \frac{1}{3}$$

$$\frac{-\frac{2}{3}}{2} \quad 3\left(x - \frac{1}{3}\right)\left(x - \frac{1}{3}\right) = \frac{37}{3}$$

$$\frac{-\frac{2}{3} \cdot \frac{1}{2}}{\left(-\frac{1}{3}\right)^2} \quad \frac{3\left(x - \frac{1}{3}\right)^2}{3} = \frac{37}{3} \cdot \frac{1}{3}$$

$$\frac{1}{9} \quad \sqrt{\left(x - \frac{1}{3}\right)^2} = \sqrt{\frac{37}{9}}$$

$$x - \frac{1}{3} = \pm \frac{\sqrt{37}}{3}$$

$$+\frac{1}{3} \quad +\frac{1}{3}$$

$$x = \frac{1}{3} \pm \frac{\sqrt{37}}{3}$$

$$\text{or} \quad x = \frac{1 \pm \sqrt{37}}{3}$$

$$\triangle + 3 = 4$$

$$\textcircled{3} + 1 = 4$$

Vertex Form

$$y = a(x - h)^2 + k$$

$$y = 2(x - 4)^2 + 3$$

Horizontal translation 4 units right

Vertical translation 3 units up

Vertical stretch by a factor of 2

Vertex (4, 3)

$$y = x^2 + 4x - 1$$

$$y + 1 = x^2 + 4x$$

$$y + 1 + 4 = x^2 + 4x + 4$$

$$y + 5 = (x + 2)(x + 2)$$

$$y + 5 = (x + 2)^2$$

$$\rightarrow y = (x + 2)^2 - 5$$

$$y = a(x - h)^2 + k$$

Vertex $(-2, -5)$

$$y = (x + 2)^2 - 5$$

Horizontal Translation 2 units left
Vertical Translation 5 units down

Complete
the
Square

$$\frac{4}{2} = 2$$

$$2^2 = 4$$

$$y = -x^2 + 4x + 2$$

$$y - 2 = -x^2 + 4x$$

$$y - 2 = -(x^2 - 4x + 4)$$

factor
out
-1 on
right

$$\frac{-4}{2}$$

$$-2$$

$$\frac{(-2)^2}{4}$$

$$y - 2 = -(x - 2)(x - 2)$$

$$y - 2 = -(x - 2)^2 + 6$$

$$y = -(x - 2)^2 + 6$$

Horizontal translation 2 units right

Vertical translation 6 units up

Reflection across x-axis

Vertex (2, 6) Opens down
Maximum

p 204

26, 28, 32

42, 46