College Algebra
Section 2.7
Rational Functions

A rational function is one that can be written in the form

$$f(x) = \frac{p(x)}{q(x)}$$

p(x) and q(x) are polynomials and q(x) is not the zero polynomial.

The domain of a rational function of x includes all real numbers except x-values that make the denominator zero.

$f(x) = \frac{2}{x}$ Domain is all real numbers except 0

$f(x) = \frac{x+3}{x-4}$ Domain is all real numbers except 4

$x - 4 = 0$

$+4 \quad +4$

$x = 4$

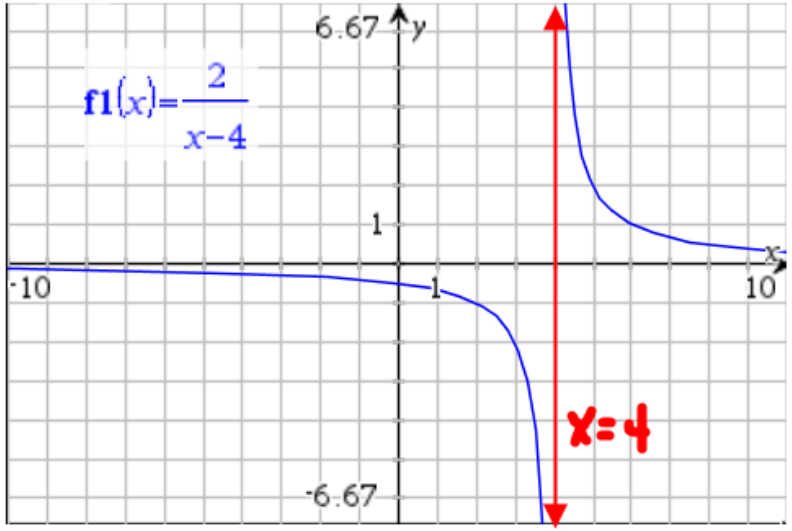
$f(4) = \frac{4+3}{4-4}$

$f(4) = \frac{7}{0}$ undefined

Vertical Asymptotes

The line $x = a$ is a vertical asymptote of the graph of f if

$f(x) \to \infty$ or $f(x) \to -\infty$

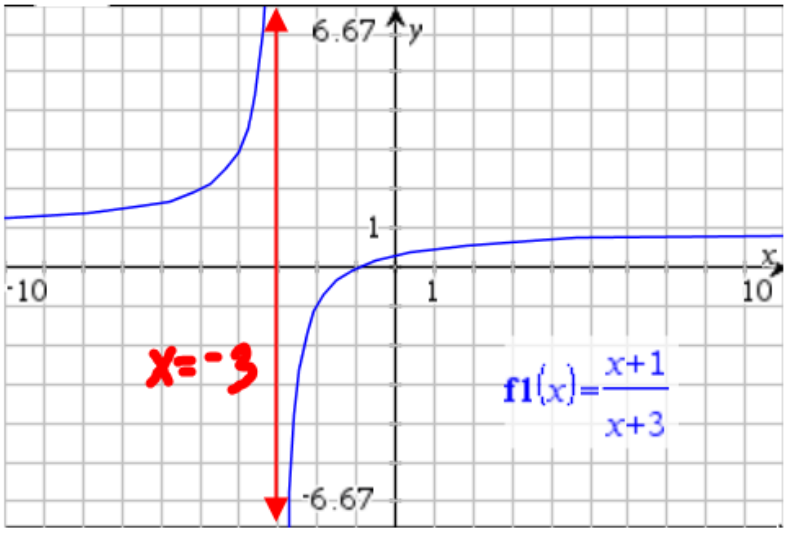


$$+4 \quad +4$$
$$y-4=0$$
$$x=4$$

$$f(x)=\frac{2}{x-4}$$

$x=4$ is a Vertical Asymptote

Domain all real numbers except 4



$$f(x)=\frac{x+1}{x+3}$$

$x = -3$ is a Vertical Asymptote

$x+3=0$
$-3 \quad -3$
$x=-3$

Domain all real numbers except -3

Horizontal Asymptotes

$$f(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + \dots a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \dots b_1 x + b_0} \qquad a_n \neq 0 \quad b_m \neq 0$$

If $n < m$, then the x-axis is a horizontal asymptote.

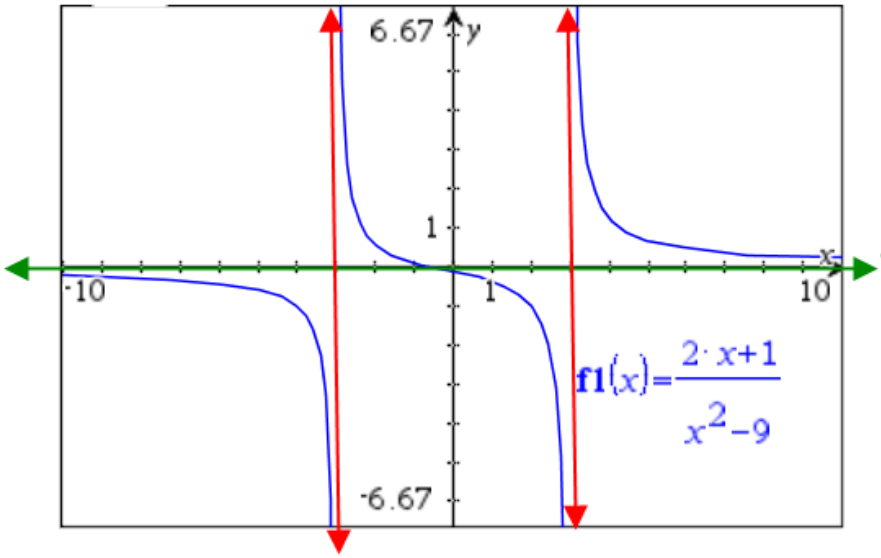
If $n = m$, then the line $y = \frac{a_n}{b_m}$ is a horizontal asymptote.

If $n > m$, then the is no horizontal asymptote.

The line $y = b$ is a horizontal asymptote of f if

$f(x) \to b$

as $x \to \infty$ or $x \to -\infty$



y=0

x=-3 x=3

$$f(x)=\frac{2x^{1}+1}{x^{2}-9}$$

1<2

n<m

The x-axis is a horizontal asymptote

y=0

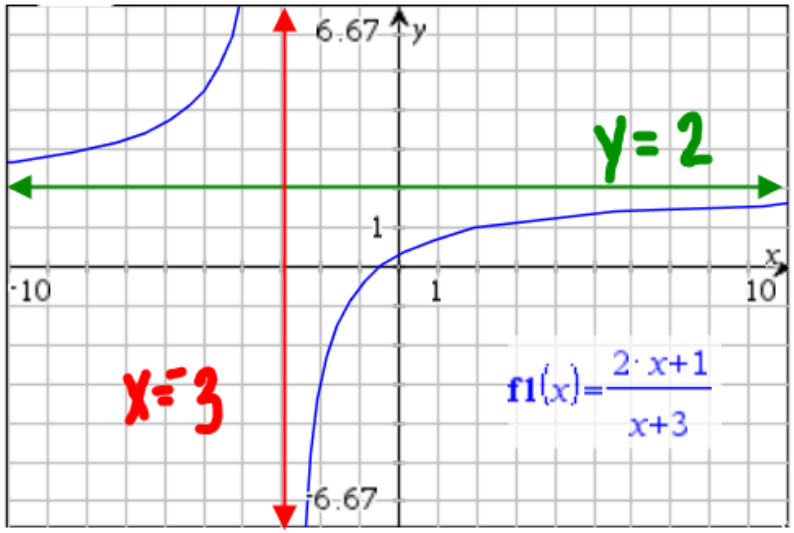
$x^2-9=0$

$(x+3)(x-3)=0$

$x+3=0$ $x-3=0$

$x=-3$ $x=3$

x=-3 and x=3 are Vertical Asymptotes



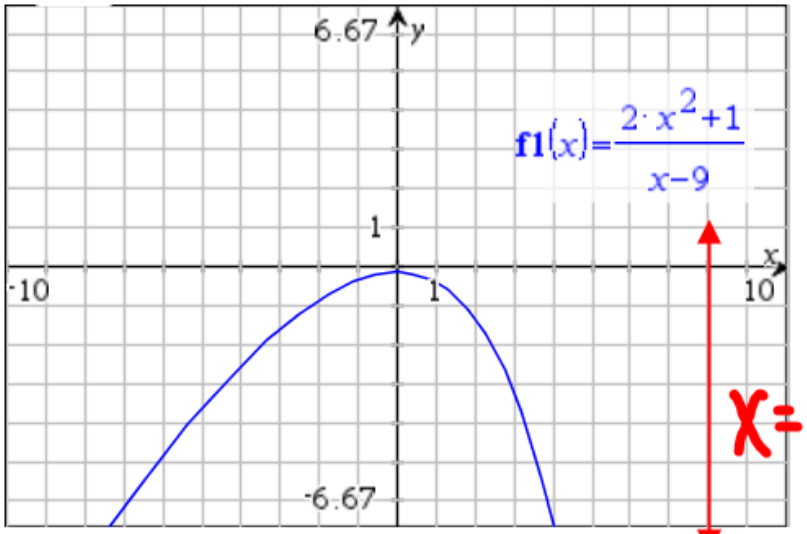
$$f(x) = \frac{2x+1}{1x+3}$$

$1 = 1$

$n = m$

The line $y = \frac{2}{1}$ or $y = 2$ is a horizontal asymptote

$x = -3$ is a vertical asymptote



$$f(x) = \frac{2x^2+1}{x^1-9}$$

$2 > 1$

$n > m$

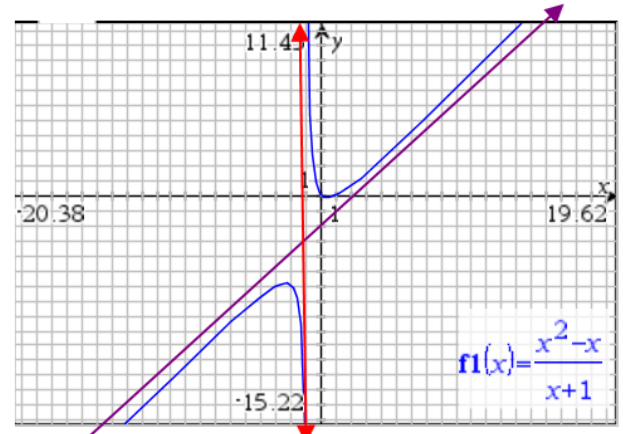
There are no horizontal asymptotes

$X=9$ is a vertical asymptote

Slant Asmyptotes

If the degree of the numerator is exactly one more than the degree of the denominator, the graph has a slant asymptote.

To find the equation of the slant asymptote, use long or synthetic division.



y = x - 2

x = -1

$f(x) = \frac{x^2 - x}{x + 1}$

Synthetic Division

$$\begin{array}{r|rrr} -1 & 1 & -1 & 0 \\ & & -1 & 2 \\ \hline & 1 & -2 & 2 \end{array}$$

$x - 2 + \frac{2}{x+1}$

Equation of the slant asymptote is $y = x - 2$