

$$31. \frac{2}{3}, -1, 3+i\sqrt{2}, 3-i\sqrt{2}$$

$$3 \cdot x = \frac{2}{3} \quad x = -1 \quad x = 3+i\sqrt{2} \quad x = 3-i\sqrt{2}$$

$$3x = 2 \quad x+1=0 \quad x-3-i\sqrt{2}=0 \quad x-3+i\sqrt{2}=0$$

$$3x-2=0$$

$$(3x-2)(x+1)(x-3-i\sqrt{2})(x-3+i\sqrt{2})=0$$

$$[3x^2+3x-2x-2] [x^2-3x+x\sqrt{2}-3x+9-3i\sqrt{2}x -x\sqrt{2}+3i\sqrt{2}x]$$

$$(3x^2+x-2)(x^2-6x+11)$$

$$\cancel{3x^4} - \cancel{18x^3} + \cancel{33x^2} + \cancel{x^3} - \cancel{6x^2} + \cancel{11x} - \cancel{2x^2} + \cancel{12x} - 22$$

$$3x^4 - 17x^3 + 25x^2 + 23x - 22$$

$$y = x^2 - 4$$

$$y = 2x^2 - 8$$

$$2.7 \quad f(x) = \frac{4}{x-3}$$

$$y = \frac{4}{x-3}$$

$$x \neq 3$$

$x=3$ Vertical Asymptote

Division by zero

$$f(x) = \frac{x^1}{x^2 - x - 2}$$

$$y = \frac{x}{(x-2)(x+1)}$$

$$x-2=0 \quad x+1=0$$

$$x=2 \quad x=-1$$

Vertical Asymptotes

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + \dots + b_1 x + b_0}$$

$n < m$ x-axis horizontal asymptote

$n = m$ $y = \frac{a_n}{b_n}$ horizontal asymptote

$n > m$ no horizontal asymptote

n degree of numerator
 m degree of denominator

$$f(x) = \frac{x^1 + 3}{x^2 + 5x + 6}$$

$n = 1$
 $m = 2$

Horizontal Asymptote $n < m$
 x -axis $1 < 2$

$$f(x) = \frac{(x+3)}{(x+2)(x+3)}$$

Vertical Asymptote.
 $x = -2$

Hole $x = -3$

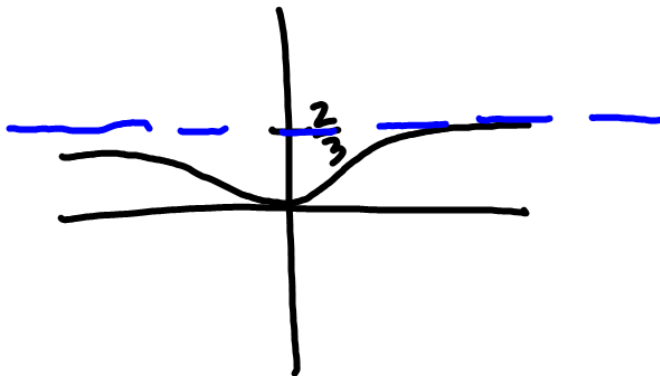
Common
 factor
 in numerator
 & denominator

$$f(x) = \frac{2x^2 + 3x + 1}{3x^2 - x - 2}$$

$$y = \frac{2}{3}$$

$$f(x) = \frac{2x^2}{3x^2 + 1}$$

$$y = \frac{2}{3}$$



Slant Asymptote

degree of numerator is
one more than the
degree of the denominator

$$f(x) = \frac{x^2 - x}{x + 1}$$

$$\begin{array}{r|rrr} -1 & 1 & -1 & 0 \\ & & -1 & 2 \\ \hline & 1 & -2 & 2 \end{array}$$

x c R

$$\boxed{x - 2} + \frac{2}{x + 1}$$

$$y = x - 2$$