3.3 Common Log $\log_{10}$
Base 10

Natural Log $\ln$
Base e

Change of Base

$$\log_a x = \frac{\log_{10} x}{\log_{10} a}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\log_3 12 = \frac{\log_{10} 12}{\log_{10} 3}$$

$3^? = 12$

or

$$\frac{\ln 12}{\ln 3}$$

Calc

2.262

$3^{2.262}$

$$\log_a(uv) = \log_a u + \log_a v$$

$$\ln(uv) = \ln u + \ln v$$

$$\log 6 \approx .778$$

$$\log_{10}(2 \cdot 3) = \log_{10} 2 + \log_{10} 3$$

$$= .301 + .477$$

$$\approx .778$$

$$\log_a \frac{u}{v} = \log_a u - \log_a v$$

$$\ln \frac{u}{v} = \ln u - \ln v$$

Ex. $\log_{10} k = .778$

$$\log_{10} \frac{18}{3} = \log_{10} 18 - \log_{10} 3$$

$$= 1.255 - .477$$

$$= .778$$

p217 37-50

$\log_b 2 = .3562$

$\log_b 3 = .5646$

$\log_b 5 = .8271$

38. $\log_b 15$

$\log_b(3 \cdot 5) = \log_b 3 + \log_b 5$

$= .5646 + .8271$

$= 1.3917$

$\log_a u^n = n \log_a u$

$\ln u^n = n \ln u$

$\log_{10} 10^5 = 5$

$5 \log_{10} 10$

$5(1) = 5$

43. $\log_b \sqrt{2}$

$\log_b 2^{\frac{1}{2}}$

$\frac{1}{2} \log_b 2$

$\frac{1}{2}(.3562)$

$.1781$

45. $\log_b 40$

$5 \cdot 8$

$5 \cdot 2^3$

$\log_b 5 \cdot 2^3$

$\log_b 5 + \log_b 2^3$

$\log_b 5 + 3 \log_b 2$

$.8271 + 3(.3562)$

$.8271 + 1.0686$

1.8957

51. $\log_4 \sqrt[3]{4}$

$\log_4 4^{\frac{1}{3}}$

$\frac{1}{3}$

66-84 p218

66. $\log_6 6x$

$\log_6 6 + \log_6 x$

$1 + \log_6 x$

68. $\log_{10} \frac{y}{2}$

$\log_{10} y - \log_{10} 2$

$\log_{10} y - .301$

12. $\ln \sqrt[3]{t}$

$\ln t^{\frac{1}{3}}$

$\frac{1}{3} \ln t$

14. $\ln \frac{xy}{z}$

$\ln x + \ln y - \ln z$

Ex. $\ln \frac{a}{bc}$

$\ln a - (\ln b + \ln c)$

85-100

85. $\log_3 x + \log_3 5$

$\log_3 x \cdot 5$

$\log_3 5x$

92. $2 \ln 8 + 5 \ln z$

$\ln 8^2 \cdot z^5$

$\ln 64 z^5$

39. $\log_b \sqrt{5b}$

$\log_b (5b)^{\frac{1}{2}}$

$\frac{1}{2}(\log_b 5 + \log_b b)$

$\frac{1}{2}(.8271 + 1)$

$\frac{1}{2}(1.827)$

.9136

3.4

Solve

$$2^x = 64$$

$$\log_2 2^x = \log_2 64$$

$$x = \log_2 64$$

Change of base

$$x = \frac{\log 64}{\log 2}$$

$$\log(64) \div \log(2)$$

$$x = 6$$

$$\log_4 x = 3$$

$$4^{\log_4 x} = 4^3$$

$$x = 64$$