

7.5 Rational Roots

Zeros Answers
x-value Solutions
when x-intercept $y=0$
 $y=0$

Rational numbers
fractions

$$\frac{1}{2} \quad \frac{3}{4} \quad \frac{5}{1} \quad -\frac{6}{1}$$

Rational Root Theorem

Example $10x^3 + 9x^2 - 19x + 6 = 0$

Factors of constant

$$p = \pm 1, \pm 2, \pm 3, \pm 6$$

Factors of the leading coefficient

$$q = \pm 1, \pm 2, \pm 5, \pm 10$$

To find possible rational roots

$$\frac{p}{q} = \frac{\pm 1}{\pm 1}, \frac{\pm 1}{\pm 2}, \frac{\pm 1}{\pm 5}, \frac{\pm 1}{\pm 10}$$

$$\begin{array}{l} \pm 1, \pm \frac{1}{2} \\ \pm \frac{1}{5}, \pm \frac{1}{10} \\ \pm 2, \pm \frac{2}{5} \\ \pm 3, \pm \frac{3}{2} \\ \pm \frac{3}{5}, \pm \frac{3}{10} \\ \pm 6, \pm \frac{6}{5} \end{array} \quad \begin{array}{l} \frac{\pm 2}{\pm 1}, \frac{\pm 2}{\pm 2}, \frac{\pm 2}{\pm 5}, \frac{\pm 2}{\pm 10} \\ \frac{\pm 3}{\pm 1}, \frac{\pm 3}{\pm 2}, \frac{\pm 3}{\pm 5}, \frac{\pm 3}{\pm 10} \\ \frac{\pm 6}{\pm 1}, \frac{\pm 6}{\pm 2}, \frac{\pm 6}{\pm 5}, \frac{\pm 6}{\pm 10} \end{array}$$

$$3x + 6 = 0$$

$$\pm 1(\pm 2), \pm 3, \pm 6$$

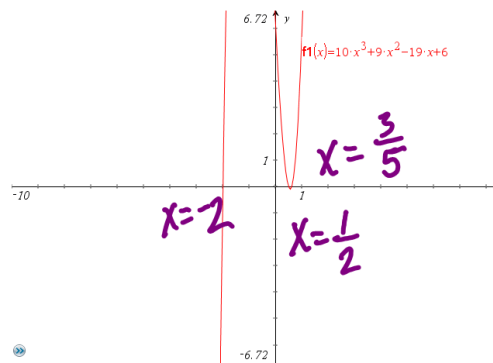
$$\pm 1, \pm 3$$

$$3x = -6$$

$$x = -2$$

$$y = 3x + 6$$

$$2y = 6x + 12$$



$$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{1}{10}, \pm 2, \pm \frac{2}{5}, \pm 3, \pm \frac{3}{5}, \pm \frac{3}{10}, \pm \frac{3}{2}, \pm 6, \pm \frac{6}{5}$$

Possible Rational Roots

$$\begin{array}{r|rrrr} -2 & 10 & 9 & -19 & 6 \\ & & -20 & 22 & -6 \\ \hline \frac{1}{2} & 10 & -11 & 3 & 0 \\ & & 5 & -3 & \\ \hline \frac{3}{5} & 10 & -6 & 0 & \\ & & 6 & & \\ \hline & 10 & 0 & & \\ & c & R & & \end{array}$$

List of Possible Rational Roots

Graph find zeros
Synthetic Division

Descartes Rule of Signs

Number of changes in sign
is the number of positive
real roots

$$+10x^3 + 9x^2 - 19x + 6 = 0$$

2 changes in sign
2 or 0 positive real roots

4 changes in sign
4, 2, 0 positive real roots
3, 1

To find the number of
negative real roots

Change x to $-x$
Substitute $-x$ in for x
Count changes in sign

$$10x^3 + 9x^2 - 19x + 6 = 0$$

$$10(-x)^3 + 9(-x)^2 - 19(-x) + 6 = 0$$

$$-10x^3 + 9x^2 + 19x + 6 = 0$$

1 change in sign
1 negative real root

Sum of Roots

$$\frac{-B}{A}$$

$$Ax^3 + Bx^{n-1} + Cx^{n-2} \dots$$

A leading coefficient

B coefficient of next term

$$10x^3 + 9x^2 - 19x + 6 = 0$$

$$A = 10 \quad \frac{-B}{A}$$

$$B = 9 \quad \frac{-9}{10}$$

$$\text{Sum of Roots} \quad \frac{-9}{10}$$

$$-2, \frac{1}{2}, \frac{3}{5}$$

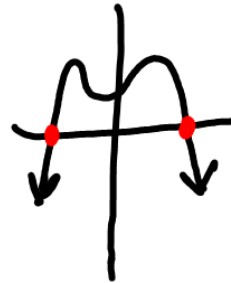
$$\frac{-20}{10} + \frac{11}{10}$$

$$\frac{-9}{10}$$

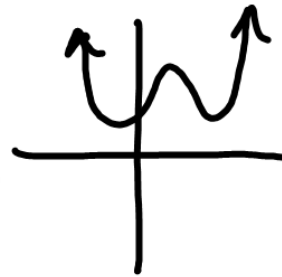
$x^4 + \dots$

4, 2, 0 pos real

2 pos. real
2 imaginary



0 real
roots



p463 12, 16, 20